Cooperative estimation and fleet reconfiguration for multi-agent systems

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Abstract: This paper considers a multi-agent system which aim is to determine the maximum of some field. For that purpose, noisy measurements are collected by each agent and exchanged between neighboring agents. The maximization task, performed by gradient climbing, has to be robust to the presence of agents equipped with sensors providing outliers. For that purpose, an outlier detection scheme is used and the optimal configuration for agents with different sensor noise characteristics is evaluated. This gives insights to derive a practical distributed control law to achieve robust maximization. The stability of the system with this control law is analyzed. The resulting performance is illustrated on an example.

Keywords: Multi agent system, cooperative estimation, fault detection, Lyapunov stability.

1. INTRODUCTION

Autonomous vehicles (moving agents) have increased ability to perform complex missions, such as exploration or surveillance of some geographical area. Such missions are more easily completed when agents cooperate (Bullo et al. (2009)). Cooperation between agents allows to use simpler sensors and vehicles and provides an increased robustness to potential failures compared to missions addressed by a single agent.

This paper considers agents equipped with sensors measuring some field (temperature, radiation, chemical agent concentration) at their location. The agents have to determine cooperatively the location of the maximum of the field over some *a priori* search zone, see Ahmadzadeh and Buchman (2006); Tang and Parker (2006); Choi and Horowitz (2007); Parker (2013). The main additional constraint considered in this paper is robustness against the presence of faulty sensors, as in Chamseddine et al. (2012).

For that purpose, each agent performs a local estimate of the field and of its gradient by sharing information over a wireless network. A control law which drives the agents towards the maximum while avoiding collisions is then evaluated as in Choi and Horowitz (2007). This approach, however, is very sensitive to erroneous measurements (outliers) potentially provided by agents equipped with faulty sensors. Such outliers may compromise the mission as shown in Zhang et al. (2010). The aim of this paper is to use Fault Detection and Identification (FDI) methods to isolate the faulty agents. It thus presents an adaptation of the control law to minimize the influence of the faulty agents on the success of the mission while keeping them in formation.

Numerous FDI methods have been presented in the literature, see, *e.g.*, Elnahrawy and Nath (2004); Jeffery et al. (2006); Janakiram et al. (2006); Wu et al. (2007); Curiac et al. (2007). For example, in Wu et al. (2007), each sensor uses the median of the measurements of its neighbors to detect possible outliers. Curiac et al. (2007) estimate the expected value of the measurement of an agent using its own previous measurements. The FDI approach presented in this paper is derived from Curiac et al. (2007) as it compares the actual measurement of an agent with its estimated value obtained from the measurements provided by the agents of its neighborhood.

Reconfiguration after fault detection is usually based on modifying the control of the agents (Zhaohui and Noura (2013)) or re-planning their trajectories, as in Chamseddine et al. (2012). The reconfiguration technique introduced in this paper modifies solely the control law of the faulty agents to limit their impact on the estimates of the field and its gradient, which reduces the computational cost.

This paper is organized as follows. First, Section 2 presents the cooperative estimation problem and the agent dynamic and measurement equations. The proposed solution, including the FDI and the optimal configuration agents should adopt is described in Section 3. A pragmatic distributed control law to drive the agents towards the field maximum is introduced in Section 4 and its stability is demonstrated. Simulations illustrate the performance of the approach in Section 5.

2. PROBLEM FORMULATION

Consider a scalar spatial field $\phi(\mathbf{x})$, defined at any position $\mathbf{x} = (x, y)^T$ of some search area $D \subset \mathbb{R}^2$. The field ϕ is assumed to be twice-continuously differentiable, time

invariant, and to have a unique maximum at some position $\mathbf{x}_M \in D$. The gradient of ϕ at $\mathbf{x}$ is

$$\nabla\phi(\mathbf{x}) = \left[\frac{\partial\phi}{\partial x}(\mathbf{x}), \frac{\partial\phi}{\partial y}(\mathbf{x})\right]^T. \qquad (1)$$

N identical agents equipped with sensors obtain measurements at discrete time instants t_k

$$y_i(t_k) = \phi(\mathbf{x}_i(t_k)) + n_i(t_k), \qquad (2)$$

of ϕ at their positions $\mathbf{x}_i(t_k)$, $i = 1,\ldots,N$. Each agent is characterized by the state $\theta_i(t_k)$ of its sensor, which may be good $\theta_i(t_k) = 0$ or defective $\theta_i(t_k) = 1$. The $\theta_i(t_k)$s are realization of time-invariant and independent Markov chains with transition probabilities for $i = 1,\ldots,N$

$$p_{01} = \Pr(\theta_i(t_k) = 1|\theta_i(t_{k-1}) = 0) \qquad (3)$$
$$p_{10} = \Pr(\theta_i(t_k) = 0|\theta_i(t_{k-1}) = 1) \qquad (4)$$

and $p_{00} = 1 - p_{01}$ and $p_{11} = 1 - p_{10}$. In (2), the $n_i(t_k)$s are realizations of independently distributed zero-mean Gaussian variables with state-dependent variance $\sigma^2_{\theta_i(t_k)}$, where $\sigma^2_0 \ll \sigma^2_1$. All agents are synchronized and make measurements at the same time. At each time instant t_k, the i-th agent is able to communicate with a subset of agents which indexes are $\mathcal{N}_i(t_k) \subset \{1,\ldots,N\}$. These communications are assumed without delay and losses.

The dynamic of each agent is modeled as

$$M\ddot{\mathbf{x}}_i + C(\mathbf{x}_i, \dot{\mathbf{x}}_i)\dot{\mathbf{x}}_i = \mathbf{u}_i \qquad (5)$$

where $\mathbf{u}_i(t_k)$ is the control input applied to agent i at time t_k, M is its mass, and $C(\mathbf{x}_i, \dot{\mathbf{x}}_i)$ a non-negative friction coefficient, see Wang (2007).

The purpose of the mission is to find

$$\mathbf{x}_M = \arg\max_{\mathbf{x} \in D} \phi(\mathbf{x}), \qquad (6)$$

while maintaining the formation, despite the presence of erroneous sensors.

3. PROPOSED SOLUTION

The proposed solution consists in four steps that will be performed during each time interval $[t_k, t_{k+1}[$. First, all agents take a measurement $y_i(t_k)$ of the field at their location $\mathbf{x}_i(t_k)$. Second, the measurement and the current agent location are broadcast to the other agents in its neighborhood. Third, using the shared measurements, all agents estimate the state of their sensor and of the sensors of their neighbors. This estimation may be performed using the various FDI techniques described in Elnahrawy and Nath (2004); Jeffery et al. (2006); Janakiram et al. (2006); Wu et al. (2007); Curiac et al. (2007). Next, each agent i performs an estimation of the field and of its gradient at the current estimate $\widehat{\mathbf{x}}_i^k$ of the location of the maximum of the field. These estimates may be different since they do not share the same information. Using gradient climbing, each agent is then able to evaluate an updated estimate $\widehat{\mathbf{x}}_i^{k+1}$. Finally, a control law is designed in a distributed way for each agent to move towards $\widehat{\mathbf{x}}_i^{k+1}$, keeping the agents in formation, while avoiding collisions, and trying to minimize the variance of the estimation error of the field and its gradient at $\widehat{\mathbf{x}}_i^{k+1}$.

In the following, we focus on the last three steps and only outline the FDI step, which is assumed successfully performed for each agent.

3.1 Field and gradient estimation

A local model of ϕ_i is derived from a second-order Taylor expansion of ϕ considered at $\widehat{\mathbf{x}}_i^k$

$$\phi_i(\mathbf{x}) = \phi(\widehat{\mathbf{x}}_i^k) + (\mathbf{x} - \widehat{\mathbf{x}}_i^k)^T \nabla\phi(\widehat{\mathbf{x}}_i^k)$$
$$+ \frac{1}{2}(\mathbf{x} - \widehat{\mathbf{x}}_i^k)^T \nabla^2\phi(\boldsymbol{\chi}_i)(\mathbf{x} - \widehat{\mathbf{x}}_i^k) \qquad (7)$$

where $\boldsymbol{\chi}_i$ belongs to the segment joining $\mathbf{x}$ and $\widehat{\mathbf{x}}_i^k$. The aim is to obtain an estimate as accurate as possible of

$$\boldsymbol{\alpha}_i^k = \begin{pmatrix} \phi(\widehat{\mathbf{x}}_i^k) \\ \nabla\phi(\widehat{\mathbf{x}}_i^k) \end{pmatrix}$$

using $y_i(t_k)$, $i = 1,\ldots,N$.

One may approximate ϕ_i in (7) as follows

$$\bar{\phi}_i(\mathbf{x}) = \phi(\widehat{\mathbf{x}}_i^k) + (\mathbf{x} - \widehat{\mathbf{x}}_i^k)^T \nabla\phi(\widehat{\mathbf{x}}_i^k), \qquad (8)$$

introducing the approximation error

$$e_i(\mathbf{x}, \widehat{\mathbf{x}}_i^k) = \phi_i(\mathbf{x}) - \bar{\phi}_i(\mathbf{x})$$
$$= \frac{1}{2}(\mathbf{x} - \widehat{\mathbf{x}}_i^k)^T \nabla^2\phi(\boldsymbol{\chi}_i)(\mathbf{x} - \widehat{\mathbf{x}}_i^k), \qquad (9)$$

corresponding to the neglected second-order term of (7).

The model (8) could be extended to take into account the Hessian matrix. However, various examples provided by Zhang and Leonard (2010) illustrate the fact that the estimation of the Hessian matrix from noisy field measurements is difficult and results in poor-quality estimates.

Using (7), Agent i models the measurement $y_j(t_k)$ provided by Agent j as follows

$$y_j(t_k) = \phi(\mathbf{x}_j(t_k)) + n_j(t_k)$$
$$= \phi(\widehat{\mathbf{x}}_i^k) + (\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k)^T \nabla\phi(\widehat{\mathbf{x}}_i^k) + n_j(t_k)$$
$$+ \frac{1}{2}(\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k)^T \nabla^2\phi(\boldsymbol{\chi}_{ij})(\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k), \qquad (10)$$

where $\boldsymbol{\chi}_{ij}$ belongs to the segment joining $\widehat{\mathbf{x}}_i^k$ and $\mathbf{x}_j(t_k)$. Then

$$y_j(t_k) = \left(1 \quad (\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k)^T\right) \boldsymbol{\alpha}_i^k$$
$$+ e_i(\mathbf{x}_j(t_k), \widehat{\mathbf{x}}_i^k) + n_j(t_k). \qquad (11)$$

Agent i collects all the measurements available in its neighborhood $N_i(t_k)$ at t_k to get

$$\mathbf{y}_{i,k} = \mathbf{R}_{i,k}\boldsymbol{\alpha}_i^k + \mathbf{n}_{i,k} + \mathbf{e}_{i,k} \qquad (12)$$

where

$$\mathbf{y}_{i,k} = \left(y_{i_1}(t_k), \ldots, y_{i_{N_i}}(t_k)\right)^T,$$

$$\mathbf{R}_{i,k} = \begin{pmatrix} 1 & (\mathbf{x}_{i_1}(t_k) - \widehat{\mathbf{x}}_i^k)^T \\ \vdots & \vdots \\ 1 & (\mathbf{x}_{i_{N_i}}(t_k) - \widehat{\mathbf{x}}_i^k)^T \end{pmatrix}, \qquad (13)$$

$$\mathbf{n}_{i,k} = \left(n_{i_1}(t_k), \ldots, n_{i_{N_i}}(t_k)\right)^T,$$

and

$$\mathbf{e}_{i,k} = \begin{pmatrix} \frac{1}{2}(\mathbf{x}_{i_1}(t_k) - \widehat{\mathbf{x}}_i^k)^T \nabla^2\phi(\boldsymbol{\chi}_{i1})(\mathbf{x}_{i_1}(t_k) - \widehat{\mathbf{x}}_i^k) \\ \vdots \\ \frac{1}{2}(\mathbf{x}_{i_{N_i}}(t_k) - \widehat{\mathbf{x}}_i^k)^T \nabla^2\phi(\boldsymbol{\chi}_{iN})(\mathbf{x}_{i_{N_i}}(t_k) - \widehat{\mathbf{x}}_i^k) \end{pmatrix} \qquad (14)$$

with $\mathcal{N}_i(t_k) = \{i_1, \ldots, i_{N_i}\}$. The measurement noise vector $\mathbf{n}_{i,k}$ is zero-mean Gaussian with diagonal covariance matrix

$$\boldsymbol{\Sigma}_n = \mathrm{diag}\left(\sigma^2_{\theta_{i_1}(t_k)}, \ldots, \sigma^2_{\theta_{i_{N_i}}(t_k)}\right). \qquad (15)$$

In absence of $\mathbf{e}_{i,k}$, the maximum likelihood estimate of $\boldsymbol{\alpha}_i^k$ would correspond to the argument of the minimum of

$$J_0(\boldsymbol{\alpha}) = (\mathbf{y}_{i,k} - \mathbf{R}_{i,k}\boldsymbol{\alpha})^{\mathrm{T}} \boldsymbol{\Sigma}_n^{-1} (\mathbf{y}_{i,k} - \mathbf{R}_{i,k}\boldsymbol{\alpha}). \qquad (16)$$

Accounting for the impact of $\mathbf{e}_{i,k}$ is more complicated. The j-th component of $\mathbf{e}_{i,k}$ is a function of $\left\|\mathbf{x}_j(t_k) - \widehat{\mathbf{x}}_i^k\right\|_2^2$, where $\|\cdot\|_2$ is the Euclidian norm. The model error grows thus quadratically with the distance between $\mathbf{x}_j(t_k)$ and $\widehat{\mathbf{x}}_i^k$. Agents which are far from $\widehat{\mathbf{x}}_i^k$ should thus have less impact on the estimate of $\boldsymbol{\alpha}_i^k$ than agents close to $\widehat{\mathbf{x}}_i^k$. The following weight matrix, close to the one used in Ogren et al. (2004), was chosen to account for both the measurement noise and the modeling error,

$$\mathbf{W}_{i,k} = \mathrm{diag}\left(\sigma^{-2}_{\theta_1(t_k)} \exp\left(\frac{-\|\mathbf{x}_1(t_k) - \widehat{\mathbf{x}}_i^k\|_2^2}{k_{\mathrm{w}}}\right), \ldots,\right.$$
$$\left.\sigma^{-2}_{\theta_N(t_k)} \exp\left(\frac{-\|\mathbf{x}_N(t_k) - \widehat{\mathbf{x}}_i^k\|_2^2}{k_{\mathrm{w}}}\right)\right), \qquad (17)$$

where k_{w} is some tuning parameter to be adjusted depending on the spatial correlation of ϕ. The weighted least-square estimate of $\boldsymbol{\alpha}_i^k$ with weighting matrix $\mathbf{W}_{i,k}$ is then

$$\widehat{\boldsymbol{\alpha}}_i^k = \left(\mathbf{R}_{i,k}^{\mathrm{T}} \mathbf{W}_{i,k} \mathbf{R}_{i,k}\right)^{-1} \mathbf{R}_{i,k}^{\mathrm{T}} \mathbf{W}_{i,k} \mathbf{y}_{i,k}. \qquad (18)$$

3.2 Bank of residuals for fault detection and identification

Model-based fault detection and identification (Curiac et al. (2007)) uses a model to predict the expected field value, which can then be compared to the actual measurement of an agent to generate a residual. This residual should be close to zero or stay between bounds when there is no fault and become large when a fault occurs.

A bank of filters is used here to identify which sensor provides a faulty measurement (if any). For the i-th sensor, $|\mathcal{N}_i(t_k)|$ residuals $r_{i,k}$ are built by excluding the k-th measurement from the estimation (18), for $k = 1, \ldots, |\mathcal{N}_i(t_k)|$.

$$r_{i,k} = (\mathbf{y}_{i,k} - \mathbf{R}_{i,k}\boldsymbol{\alpha}). \qquad (19)$$

By design, $r_{i,k}$ is sensitive to faults on all sensors, except the one affecting the k-th sensor (this is usually named as a *generalized filter scheme*). Since $r_{i,i}$ includes its own measurement y_i, it remains sensitive to a fault on the i-th sensor and is therefore sensitive to all faults. It can be used as a detection signal only, and the $|\mathcal{N}_i(t_k)| - 1$ other residuals can be used only when $r_{i,i}$ raises an alarm so as to limit the computational load of the method.

At every time step, each sensor updates a list of sensors that it considers as faulty. A consensus on the possible faulty sensor is then obtained on the fleet as follows: each sensor broadcasts the list of sensors that it has found to be faulty using its bank of filters. The one which has been voted most often is declared to be faulty, and reconfiguration can be sought for to limit its contribution to the estimation.

3.3 Updated estimate of the location of the field maximum

A new estimate $\widehat{\mathbf{x}}_i^{k+1}$ of the location of the field maximum is evaluated from $\widehat{\mathbf{x}}_i^k$ and $\widehat{\boldsymbol{\alpha}}_i^k$. For that purpose, one has first to evaluate whether $\widehat{\mathbf{x}}_i^k$ actually corresponds to an increase of ϕ compared to the value that has been obtained for $\widehat{\mathbf{x}}_i^{k-1}$. Let λ_i^k be the gradient step size at time t_k. One updates λ_i^{k-1} as follows

$$\lambda_i^k = \begin{cases} \min\left\{\lambda_{\max}, 2\lambda_i^{k-1}\right\} & \text{if } \widehat{\phi}\left(\widehat{\mathbf{x}}_i^k\right) > \widehat{\phi}\left(\widehat{\mathbf{x}}_i^{k-1}\right), \\ \lambda_i^{k-1}/4 & \text{else,} \end{cases} \qquad (20)$$

where $\lambda_{\max}$ is a fraction $\beta \in [0,1[$ of the maximum displacement an agent can perform during a time slot.

Using gradient climbing, one then gets

$$\widehat{\mathbf{x}}_i^{k+1} = \widehat{\mathbf{x}}_i^k + \lambda_i^k \widehat{\nabla\phi}\left(\widehat{\mathbf{x}}_i^k\right) / \left\|\widehat{\nabla\phi}\left(\widehat{\mathbf{x}}_i^k\right)\right\|_2. \qquad (21)$$

The classical step-size adaptation scheme (20), see, *e.g.*, Walter (2014), enables the agents to slow down when reaching the global maximum of the field ϕ.

3.4 Optimal agent configuration

The control law for the i-th agent has to be such that the agents remain in formation, avoid collisions, and minimize the variance of the estimation error of $\boldsymbol{\alpha}_i^{k+1}$ at $\widehat{\mathbf{x}}_i^{k+1}$. From (18), one may deduce an approximation [1] of the covariance of $\widehat{\boldsymbol{\alpha}}_i^{k+1}$ at $\widehat{\mathbf{x}}_i^{k+1}$

$$\widehat{\boldsymbol{\Sigma}}_{\alpha_i^{k+1}} = \left(\mathbf{R}_{i,k+1}^{\mathrm{T}} \mathbf{W}_{i,k+1} \mathbf{R}_{i,k+1}\right)^{-1}. \qquad (22)$$

To get a small $\widehat{\boldsymbol{\Sigma}}_{\alpha_i^{k+1}}$, one chooses to determine the target position of each agent that maximizes the trace of $\mathbf{R}_{i,k+1}^{\mathrm{T}} \mathbf{W}_{i,k+1} \mathbf{R}_{i,k+1}$ under the constraint that it does not collide with any other agents at t_{k+1}. This is translated in the following constrained optimization problem

$$(\mathbf{x}_i(t_{k+1}) \ldots \mathbf{x}_N(t_{k+1})) =$$
$$\arg \max_{(\mathbf{x}_1, \ldots, \mathbf{x}_N)} \mathrm{tr}\left(\mathbf{R}_{i,k+1}^{\mathrm{T}} \mathbf{W}_{i,k+1} \mathbf{R}_{i,k+1}\right) \qquad (23)$$
$$\text{st } \|\mathbf{x}_i - \mathbf{x}_j\|_2^2 \geqslant \delta^2, \ j > i.$$

To solve this problem, one introduces the Lagrangian associated to (23) and uses (13) and (17)

$$\mathcal{L}(\mathbf{x}_1, \ldots, \mathbf{x}_N, \boldsymbol{\mu}) = \sum_{i=1}^{N} \sigma^{-2}_{\theta_i(t_{k+1})} \exp\left(\frac{-\|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\|_2^2}{k_{\mathrm{w}}}\right)$$
$$\cdot \left(1 + \|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\|_2^2\right) + \sum_{j>i} \mu_{ij} \|\mathbf{x}_i - \mathbf{x}_j\|_2^2. \qquad (24)$$

where the $\mu_{i,j}$s are Lagrange multipliers. Taking the partial derivatives of (24) with respect to $\mathbf{x}_i$, one gets

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_i} = 2\sigma^{-2}_{\theta_i(t_{k+1})} \left(\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\right) \exp\left(\frac{-\|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\|_2^2}{k_{\mathrm{w}}}\right)$$
$$\left(1 - \frac{1}{k_{\mathrm{w}}}\left(1 + \|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\|_2^2\right)\right) + 2\sum_{j\neq i} \mu_{ij}(\mathbf{x}_i - \mathbf{x}_j) \quad (25)$$

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[1] $\widehat{\boldsymbol{\alpha}}_i^{k+1}$ is assumed unbiased, even if it is not the case in general, due to the presence of $\mathbf{e}_{i,k}$. Close to $\mathbf{x}_M$, more specifically, the components of $\mathbf{e}_{i,k}$ are likely to be negative.

Assuming first that $\mu_{ij} = 0$ for all $i \neq j$ one may easily show that one should have

$$\left\| \mathbf{x}_i\left(t_{k+1}\right) - \widehat{\mathbf{x}}_i^{k+1} \right\|_2^2 = k_{\mathrm{w}} - 1$$

which is possible only provided that $k_{\mathrm{w}} > 1$. In this case, $\mathbf{x}_i\left(t_{k+1}\right)$ has to be located on a circle of radius $\sqrt{k_{\mathrm{w}} - 1}$ centered in $\widehat{\mathbf{x}}_i^{k+1}$. A necessary condition for all agents to coexist on this circle while complying with the constraint of (23) is

$$2\pi\sqrt{k_{\mathrm{w}} - 1} > N\delta. \qquad (26)$$

The condition $k_{\mathrm{w}} > 1$ corresponds to a modeling error increasing slowly with the distance to the point where the Taylor expansion has been performed, which is satisfied when ϕ varies slowly.

Assume now that $\mu_{ij} \neq 0$ for some $j \neq i$. Then, at t_{k+1}, the $\mathbf{x}_i$s have to satisfy for $i = 1, \ldots, N$

$$\sigma_{\theta_i(t_{k+1})}^{-2}\left(\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\right)\exp\left(-\frac{\left\|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\right\|_2^2}{k_{\mathrm{w}}}\right)$$
$$\cdot\left(1 - \frac{1 + \left\|\mathbf{x}_i - \widehat{\mathbf{x}}_i^{k+1}\right\|_2^2}{k_{\mathrm{w}}}\right) + \sum_{j \neq i}\mu_{ij}\left(\mathbf{x}_i - \mathbf{x}_j\right) = 0. \quad (27)$$

The general case is difficult to solve. In the case of two agents, introducing

$$\boldsymbol{\delta}_1 = \mathbf{x}_1\left(t_{k+1}\right) - \widehat{\mathbf{x}}^{k+1}$$

and

$$\boldsymbol{\delta}_2 = \mathbf{x}_2\left(t_{k+1}\right) - \widehat{\mathbf{x}}^{k+1},$$

one may show (details are omitted due to lack of space) that

- when $\sigma_{\theta_1(t_{k+1})}^2 = \sigma_{\theta_2(t_{k+1})}^2$, necessarily, $\boldsymbol{\delta}_1 = -\boldsymbol{\delta}_2$ and $\|\boldsymbol{\delta}_1\|_2 = \delta/2$,
- when $\sigma_{\theta_1(t_{k+1})}^2 \ll \sigma_{\theta_2(t_{k+1})}^2$, $\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$ should still be colinear with $\|\boldsymbol{\delta}_1\|_2 \ll \|\boldsymbol{\delta}_2\|_2$ and $\|\boldsymbol{\delta}_1\|_2 + |\boldsymbol{\delta}_2\|_2 = \delta$.

4. CONTROL LAW WITH POSSIBLE RECONFIGURATION

Section 3.4 provides some insights on the way the agents should evolve to fulfill the mission described in Section 2. When k_{w} is larger than 1 and when N is small enough to satisfy (26), the control law of the agents should be such that they move on a circle of radius $\sqrt{k_{\mathrm{w}} - 1}$ centered in $\widehat{\mathbf{x}}_i^{k+1}$. This result is obtained whatever the state of their sensors. When k_{w} is smaller than 1, or when N is too large, the agents with sensors in good state should be closer to $\widehat{\mathbf{x}}_i^{k+1}$ than those with defective sensors.

4.1 Proposed control law

In what follows, we assume that the update of the estimate $\widehat{\mathbf{x}}_i$ is performed at a frequency large enough to consider it as a twice-continuously differentiable function $\widehat{\mathbf{x}}_i(t)$ of t.

Each agent is controlled *independently* of the other agents and only requires the knowledge of the position of its neighbors for collision avoidance. The proposed control law assumes further that $k_{\mathrm{w}} \leqslant 1$ or that $N \geqslant 2\pi\sqrt{k_{\mathrm{w}} - 1}$, so that agents with good sensors have to be located closer to

$\widehat{\mathbf{x}}_i(t)$ than agents with bad sensors. The structure of the control law is inspired from that of Cheah et al. (2009)

$$u_i = M\ddot{\widehat{\mathbf{x}}}_i + C(\mathbf{x}_i, \dot{\mathbf{x}}_i)\dot{\mathbf{x}}_i - k_1\left(\dot{\mathbf{x}}_i - \dot{\widehat{\mathbf{x}}}_i\right)$$
$$+ 2k_2\sum_{j=1}^{N}\left(\mathbf{x}_i - \mathbf{x}_j\right)\frac{g_{ij}}{q} - k_3^i(\theta_i)(\mathbf{x}_i - \widehat{\mathbf{x}}_i), \qquad (28)$$

where $k_1 > 0$ is used to adapt the speed of each agent to the speed of $\widehat{\mathbf{x}}_i$. The constant $k_2 > 0$ determines the relative importance of the collision avoidance term in (28), where

$$g_{ij} = \exp\left(-\boldsymbol{\delta}_{ij}^T\boldsymbol{\delta}_{ij}/q\right), \qquad (29)$$

with $\boldsymbol{\delta}_{ij} = \mathbf{x}_i - \mathbf{x}_j$, the difference of position between agents i and j, with q a function of the square of the minimum safety distance between agents. Finally, $k_3^i(\theta_i) > 0$ determines the attractivity of $\widehat{\mathbf{x}}_i$.

4.2 Reconfiguration

As indicated in Section 3.4, agents with bad sensors should be driven farther away from $\widehat{\mathbf{x}}_i$ than agents with good sensors. Such a behavior is obtained by modifying the value of gain $k_3^i(\theta_i)$.

To analyze the effect of a change of $k_3^i(\theta_i)$, consider first the fleet at equilibrium, with all sensors in good state. At equilibrium, (5) combined with (28) becomes for the i-th agent

$$-k_3^i(\mathbf{x}_i - \widehat{\mathbf{x}}_i^k) + 2k_2\sum_{j \neq i}(\mathbf{x}_i - \mathbf{x}_j)\frac{g_{ij}}{q} = 0. \qquad (30)$$

After some manipulations, (30) may be rewritten as

$$\mathbf{x}_i - \widehat{\mathbf{x}}_i^k = \frac{2k_2}{2k_2\sum_{j \neq i}\frac{g_{ij}}{q} - k_3^i(\theta_i)}\sum_{j \neq i}(\mathbf{x}_j - \widehat{\mathbf{x}}_i)\frac{g_{ij}}{q}. \quad (31)$$

Now, assume that at a given time instant, the i-th sensor becomes defective and has been identified as such. Assuming that the positions of the other agents are not significantly affected by the modification of θ_i, $\sum_{j \neq i}(\mathbf{x}_j - \widehat{\mathbf{x}}_i)\frac{g_{ij}}{q}$ is approximately constant. To drive the i-th sensor away from $\widehat{\mathbf{x}}_i$, one has to ensure that the absolute value of

$$\gamma^i(\theta_i) = \frac{2k_2}{2k_2\sum_{j \neq i}\frac{g_{ij}}{q} - k_3^i(\theta_i)} \qquad (32)$$

when $\theta_i = 1$ is larger than its absolute value when $\theta_i = 0$. This is performed by appropriately modifying the value of $k_3^i(\theta_i)$.

4.3 Stability analysis

Consider the candidate Lyapunov function

$$V = \frac{1}{2}\sum_{i=1}^{N}\left((\dot{\mathbf{x}}_i - \dot{\widehat{\mathbf{x}}}_i^k)^T M(\dot{\mathbf{x}}_i - \dot{\widehat{\mathbf{x}}}_i^k)\right.$$
$$\left. + (\mathbf{x}_i - \widehat{\mathbf{x}}_i^k)^T k_3^i(\theta_i)(\mathbf{x}_i - \widehat{\mathbf{x}}_i^k) + k_2\sum_{j=1}^{N}g_{ij}\right). \qquad (33)$$

Assume that θ_i is constant for each sensor. After some derivations following those in Cheah et al. (2009) and not

detailed here due to lack of space, one shows that the time derivative of V satisfies

$$\dot{V} = \sum_{i=1}^{N} \left[-k_1 (\dot{\mathbf{x}}_i - \dot{\hat{\mathbf{x}}}_i^k)^T (\dot{\mathbf{x}}_i - \dot{\hat{\mathbf{x}}}_i^k) \right] < 0. \qquad (34)$$

The time derivative $\dot{V}$ of V for the designed control law thus ensures the global asymptotic stability of the system.

When the θ_is are not constant, assuming that $p_{10} = 0$, *i.e.*, that a sensor does not return to the good state once it is defective, the system will jump from one globally stable configuration to another globally stable configuration. Since the number of jumps is limited to N, the number of agents, the system remains globally stable.

Finally, the control law ensures that the speed of each agent will asymptotically converge to the speed of the estimate of the field. Moreover, all agents will move closer to $\hat{\mathbf{x}}_i$, while avoiding collision. Note that, for faulty agents, the movement towards $\hat{\mathbf{x}}_i$ will be counteracted by the collision avoidance scheme.

5. SIMULATION EXAMPLE

A fleet of $N = 15$ moving agents is considered. All agents are assumed to communicate with each other, *i.e.*, $\mathcal{N}_i(t) = \{1, \ldots, N\}$ for all i and all t. Agents share their positions, their present measurements, and their estimated location of the maximum $\hat{\mathbf{x}}_i^k$. From these data, each vehicle computes its own control law.

The FDI is assumed to be performed efficiently by each agent, so that it can adjust the value of its gain k_3^i. The gradient estimation at $\hat{\mathbf{x}}_i^k$ can be done either by all the agents with their information, or an operator or a single agent that transmits the result to the others. The variance of the measurement noise of good and defective sensors is taken respectively as $\sigma_0^2 = 10^{-3}$ and $\sigma_1^2 = 10^{-1}$.

5.1 Reconfiguration is necessary

To illustrate the necessity of an adaptation of the gains k_3^i, leading to a reconfiguration of the fleet, the agents are first placed in a Gaussian field centered in $(10, 35)^T$ with covariance matrix $\sigma^2 \mathbf{I} = 5.10^{-3}\mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

The agents were initially randomly placed around $\hat{\mathbf{x}}^0 = (45, 15)^T$. The other parameters for the simulation are $M = 1$, $C = 0.01$ for the dynamic of the agents, $k_1 = 50$, $k_2 = 50$, $q = 0.1$. The measurement period is 0.1 s. In this simulation, k_3 is constant and chosen equal to 1500.

Figure 1 shows the evolution with time of the absolute value of the difference between the estimated gradient and the real gradient of the field. The red line refers to a formation with an agent with defective sensor in the middle of the fleet, while the blue line is for an agent with defective sensor at its border.

This confirms, as expected, the fact that an agent with a defective sensor should be driven farther away from $\hat{\mathbf{x}}_i(t)$ than agents with good sensors.
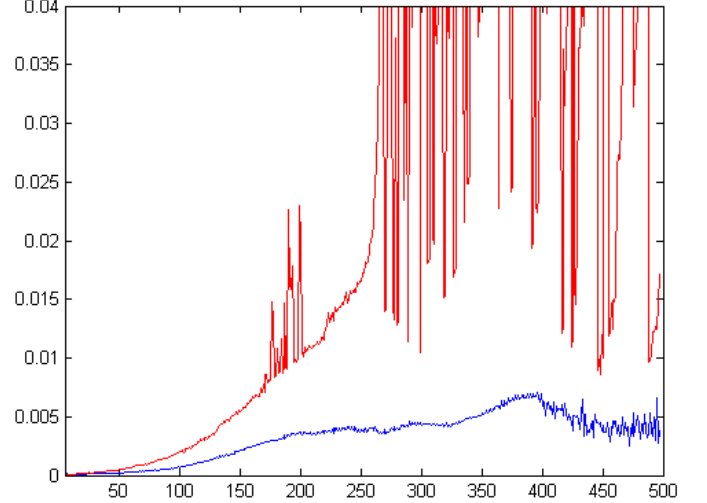

Figure 1. Evolution with time of the absolute value of the difference between the estimated gradient and the real gradient of the field when and agent with defective sensor is in the middle of the fleet (red) and on the border of the fleet (blue).

5.2 Field maximization

Consider now a Gaussian field ϕ with maximum $\mathbf{x}_M = (10, 35)^T$ and covariance matrix $\sigma^2 \mathbf{I} = 10^{-3}\mathbf{I}$. The agents are randomly placed around $\hat{\mathbf{x}}^0 = (40, 7)^T$. The parameters are as in the previous section, except for k_3, which is now adapted. The number of closest neighbors for a given agent being between 1 and 6, one chooses $k_3(0) = 1500$ and $k_3(1) = 10$, which satisfy the condition $|\gamma^i(0)| > |\gamma^i(1)|$.

The localization of the field maximum by the agents is illustrated in Figure 2. The agents with good sensors are plotted with green dots. Those with defective ones are in black.
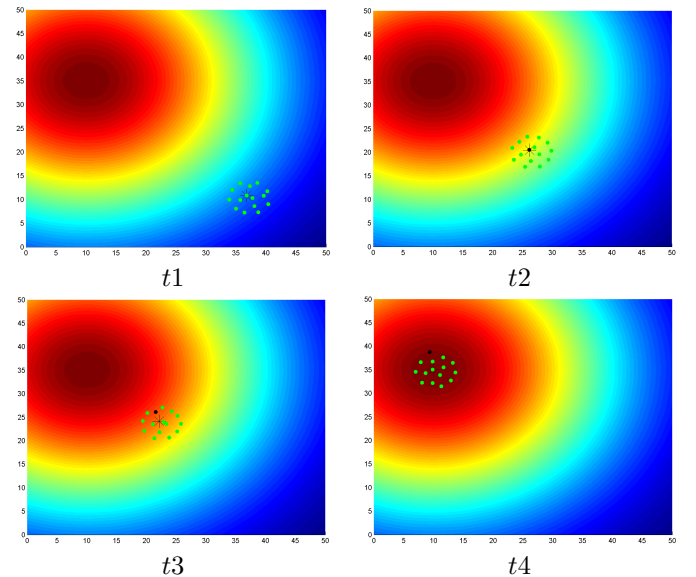

Figure 2. Maximum seeking of a field with fleet reconfiguration

At time t_1 the agents are in formation around the position $\widehat{\mathbf{x}}(t_1)$ and the fleet starts to move along the direction of the gradient. At time t_2, a faulty agent is detected by the FDI scheme. The gain k_3 of the agent with defective sensor is set to $k_3(1)$, leading to a reconfiguration of the fleet. At time t_3, the reconfiguration process is in progress, the defective agent reaches the border of the fleet. The influence of the measurement of this agent on the estimation of the gradient is thus significantly decreased. The last time instant t_4 illustrates how the formation reaches the maximum of the field.

6. CONCLUSIONS

This paper presents a distributed field maximization technique using a fleet of mobile agents. To improve robustness against measurement outliers provided by defective sensors, the configuration minimizing the covariance matrix of the estimate of the field and of its gradient in the presence of inhomogeneous measurement noise is evaluated for two sensors. This provides insights on the way a fleet reconfiguration should be performed when outliers are detected. The proposed distributed control law allows each agent to find autonomously its best location in the fleet.

The final version of the paper will provide further experimental studies for more complex fields and with an increased number of defective sensors.

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